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COLLECTIVE ENERGY DISSIPATION AND FLUCTUATIONS IN ELASTOPLASTIC SYSTEMS

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Starting from the equations of motion of a simple system possessing the properties of elastic and plastic bodies, we reconstruct its Lagrangian and Hamiltonian functions and also the so-called Rayleigh dissipation function. This allows us to find the rate of the system «heating» and to analyse the fluctuations of the basic observables. In this way a rather general scheme of solving analogous problems in more complex elastoplastic systems is established.

The paper gives a basis for studying open problems in the nuclear fusion and heavy-ions quasi-elastic collisions processes. It may be applied also for the theoretical treatment of dynamical problems in the other mesoscopic systems of fermions.

The investigation has been performed at the Bogoliubov Laboratory of Theoretical Physics and Laboratory of Nuclear Problems, JINR in collaboration with INFN (Catania, Italy).

Диссипация коллективной энергии и флуктуации в эластопластических системах

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Беря за основу уравнения движения простой системы, обладающей свойствами упругих и пластических тел, мы восстанавливаем ее функции Лагранжа и Гамильтона, а также диссипативную функцию Рэлея. Это позволяет нам найти скорость «нагревания» системы и проанализировать флуктуации основных наблюдаемых. Таким образом устанавливается достаточно общая схема решения аналогичных проблем в более сложных эласто-пластических системах.

В работе представлен метод изучения проблем, возникающих в процессах слияния ядер и квазиупругих столкновений тяжелых ионов. Он может быть также использован для теоретического описания динамических задач в некоторых других системах, состоящих из большого, но конечного числа частиц.

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1. Introduction

Elastoplastic materials are well known in nature. Their name signifies that these materials combine properties of elastic and plastic bodies. As the first, they respond with an elastic force restoring and initial configuration when they are brusquely disturbed, but as plastic materials they easily change their shape under an external pressure.

The mathematical studies of elastoplastic materials date to the time of Maxwell. Most of such studies concern macroscopic bodies for which the plasticity is a well understood property. However, for about a decade one discusses in the literature the elastoplastic properties of atomic nuclei [1], [2]. The microscopic (quantum) nature of nuclei demands some additional formal study of such systems. A study of this kind is presented in this paper on the basis of very simple equations of motion describing an elastoplastic dynamics.

The model considered below is similar but much simpler than the models describing nuclear elasto-plasticity. However, it allows some qualitative comparison with more realistic nuclear models and may be eventually generalized to study the nuclear processes in a quantitative way. One may think also that algorithms formulated in a way which is free from complications of a specific model may be used outside the realm of the nuclear physics, probably in the molecular physics and in the physics of atomic clusters.

The paper is organized as follows:

— In **Section 2** we give a general description of elastoplastic systems and present the equations of motion of a simple elastoplastic system which will be subjected for examination. The model is compared with the one describing collective dynamics in the nuclear fusion process.

— In **Section 3** we give a Rayleigh-Lagrange form for the equations of motion.

— In **Section 4** we define the collective Hamiltonian and study the rate of the collective energy dissipation.

— **Section 5** is devoted to the study of the fluctuations of the collective energy in the system analysed before. The Langevin-type equations are formulated for collective variables. We find that the fluctuations of some of collective variables are suppressed at the beginning of regaining the equilibrium state.

— We finish the paper with a short conclusion.

2. Elasto-Plastic Systems and Nuclei

Consider a system described by the equations

$$\frac{1}{2}\ddot{Q} + \frac{\alpha}{2}Q = \Pi, \tag{1}$$

$$\dot{\Pi} + \frac{\beta}{2}\dot{Q} = -\frac{1}{\tau}\Pi. \tag{2}$$

The right-hand side of Eq.(1) plays a role of a part of the force affecting the physical quantity $Q(t)$. The second of these equations may be transformed to an integral form giving

$$\Pi(t) = -\frac{\beta}{2}\int_{t_0}^{t} dt' \exp\left(-\frac{t-t'}{\tau}\right)\dot{Q}(t') + C\exp\left(-\frac{t}{\tau}\right) \tag{3}$$

and showing that this part of the force is determined by the evolution of the system during the preceding period of time of the order of τ. One may say that the system «decides» what to do in the next moment recollecting information on what had happened to it before. For this reason this system could be called also as a system with a «memory». Thus, the parameter τ determines the memory scale. In the processes going slowly in this scale (in adiabatic processes) the system described by Eqs.(1), (2) follows in its evolution an equation of motion of a vibrator with the frequency $\sqrt{\alpha}$ $(\alpha > 0)$ damped by the friction force $\chi\dot{Q}(t)$ where $\chi = \tau\beta/2$:

$$\frac{1}{2}\ddot{Q} + \chi\dot{Q} + \frac{\alpha}{2}Q = 0. \tag{4}$$

When $\chi^2 \geq \alpha$, vibrations are overdamped, and the body is plastic.

The fast (diabatic) processes in the same system proceed as if the system were an elastic body (see Ref.2):

$$\ddot{Q} + (\alpha + \beta)Q = 0.$$

In this regime the vibrational frequency is renormalized and is equal to $\Omega = \sqrt{\alpha + \beta}$.

Choosing an appropriate time scale one may always transform the parameters in Eq.(1) and Eq.(2) in such a way that $|\alpha| = 1$. The elastoplastic properties are then well pronounced when $\beta >> 1$. The «memory» scale parameter τ determines the division of the perturbations into the slow and the fast ones: subjected to fast perturbations the system reacts as an elastic body, in the other case — as a plastic one.

The evolution of the system described by Eqs.(1),(2) is demonstrated in Fig.1 and Fig.2. Here the time dependence of Q and Π variables is shown for a «large» value of β

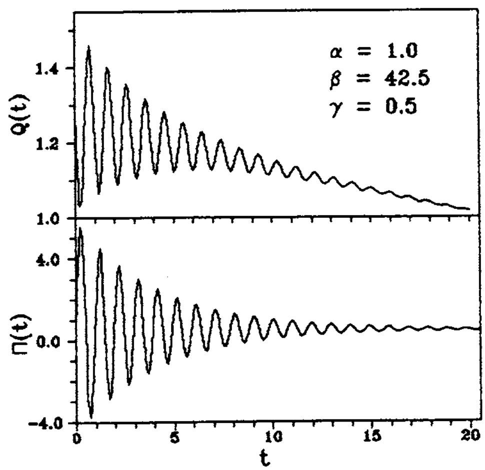

Fig.1. Above: time dependence of Q variable; below: time dependence of Π variable for $\alpha = 1$, $\beta = 42.5$ and $\gamma = 0.5$

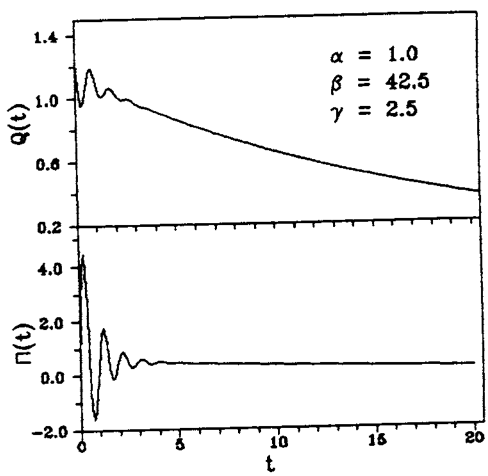

Fig.2. Same as in Fig.1 for $\alpha = 1$, $\beta = 42.5$ and for $\gamma = 2.5$

and for two different values of $\gamma = \tau^{-1}$. Elastic properties are more pronounced for small values of γ, but are distinctly seen even when it becomes comparable with the «diabatic» vibrational frequency $\sqrt{\beta}$.

Regarding the nonzero starting values of Q and $\dot{Q}(t)$ variables as a result of instantaneous external perturbation, one may say that an external force applied to such a system produces at the beginning phenomena typical for elastic bodies. The final part of the evolution process corresponds to an exponential decrease of the absolute value of the variable Q (when $\alpha > 0$). Hence, the way of coming to the equilibrium state of this system reveals its plastic properties.

Experiments using heavy ions with energies close to the Coulomb barrier performed during the last decade reveal rather unexpected nuclear properties. Depending on the experimental conditions, colliding heavy nuclei behave either as elastic bodies or as bodies made of a very plastic material. The studies of nuclear elastoplasticity have already a certain history [1], [2]. Equations (1) and (2) of the previous section have essentially the same structure as those obtained in Ref.2 for the description of the nuclear fusion. In application to the process, the elements in these equations have the following meaning:

a) The quantity $Q(t)$ is the $Q(t)_{2,0}$ component of the nuclear quadrupole mass tensor;

b) The quantity $\Pi(t)$ is the $(\lambda, \mu = 2{,}0)$ component of the intrinsic kinetic energy tensor;

c) The quantity $\alpha Q(t)$ in Eq.(1) substitutes the term in the corresponding equation considered in Ref.2 originating from the deformation dependence of the nuclear self-consistent potential $V(\mathbf{r}, t)$. The origin of the quantity $(\beta/2)\dot{Q}(t)$ in Eq.(2) lies in the coupling between the time dependent deformations of the geometrical and Fermi surfaces;

d) The parameter τ is nothing but the mean relaxation time appearing in the approximate expression for the «collision» or «relaxation» term in the microscopic theories based upon the kinetic equation.

For small Q values the model of Ref.2 gives a good description of nuclear giant quadrupole resonance [3]. The width of GQR in the daughter nucleus for the system of two fusing ^{58}Ni nuclei is well reproduced when $\gamma = h/\tau = 2.5$. The same equations [2] yield $\beta = 42.5$ for the same fusing system.

The nuclear elastoplasticity reveals itself in a number of ways. It determines the conditions for the fusion and explains the «extra push» phenomenon [4], [5]. It explains the anisotropy of γ-radiation from the low-spin fraction of fused nuclei.

The model of fusion formulated in Ref.2 allows a relatively simple numerical analysis and leads to a number of nontrivial predictions concerning the nuclear fusion. Having evident merits this model is, however, limited in its application for various reasons. One particular drawback of the model: it treats the collective processes taking place in nuclear reactions in a purely classical way. The equations of the model define a classical «trajectory» of the system which, at best, may be associated with some mean characteristics of the process. In quantal systems as nuclei, the fluctuations around the mean values play an important role.

The study of fluctuations in an elasto-plastic system is one of the subjects of this paper. There is another problem which we want to solve in this paper: the determination of the collective energy associated with the motion in the elasto-plastic system of the type described in Ref.2.

3. «Rayleigh–Lagrange» Form of Equations of Motion

To know the partition between the collective energy and the energy of intrinsic (statistical) excitation one must render the equations of motion a form as close as possible to a canonical Hamiltonian form. It turns out that it is easier to arrive first at the Lagrangian form of such equations and then to use the well-known algorithms to pass to the Hamiltonian form.

To render a canonical form to Eqs.(1),(2), let us consider the $\Pi(t)$ variable in these equations as a generalized velocity, introducing a generalized co-ordinate $Z(t)$ such that:

$$\Pi(t) = \dot{Z}(t). \tag{5}$$

Then Eqs.(1),(2) become

$$\begin{gathered} \ddot{Q} + \alpha Q - 2\dot{Z} = 0, \\ \frac{4}{\beta}\ddot{Z} + 2\dot{Q} = -4\frac{\dot{Z}}{\tau\beta}. \end{gathered} \tag{6}$$

If the right-hand side of the second of Eqs.(6) were equal to zero, these two equations would satisfy the conditions at which their standard Lagrangian formulation is possible [6]. The expression in the right-hand side could be also incorporated in the Lagrangian formulation. One may do it in the same way as one treats the friction force [6] by introducing two functions: the Lagrangian function

$$L = \frac{M_Q}{2}(\dot{Q}^2 - \alpha Q^2) + 2M_Q\left(\frac{1}{\beta}\dot{Z}^2 + \dot{Z}Q\right) \tag{7}$$

and the dissipation function

$$\Sigma = 2\frac{M_Q}{\tau\beta}\dot{Z}^2, \tag{8}$$

where M_Q is an arbitrary constant to be fixed later on.

4. Collective Energy

Now, we introduce the generalized momenta $P_i = \partial L/\partial\dot{Q}_i$ and the Hamiltonian function

$$H_{\text{coll}}(P_i, Q_i) = \sum_i P_i\dot{Q}_i - L.$$

The standard technique of transforming the Lagrange equations to the Hamiltonian form applied to the system, in which the dissipations function operates, leads to the following equations

$$\dot{Q}_i = \frac{\partial}{\partial P_i}H_{\text{coll}}, \quad \dot{P}_i = -\left(\frac{\partial\Sigma}{\partial\dot{Q}_i} + \frac{\partial}{\partial Q_i}H_{\text{coll}}\right).$$

From these equations it follows:

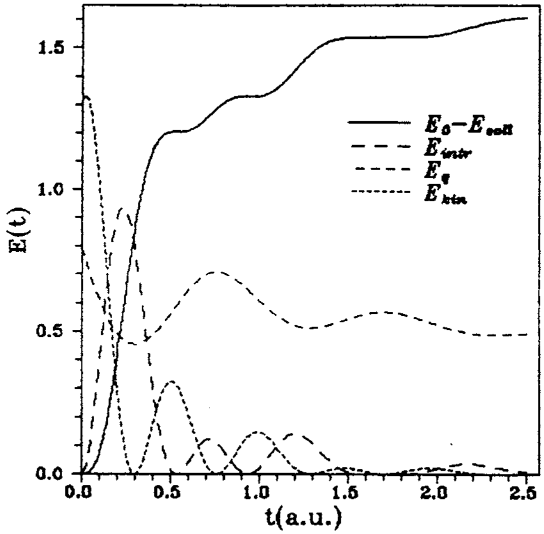

Fig.3. Partition of the total energy into different channels: 1) thermal energy $E_0 - E_{\text{coll}}$, 2) intrinsic collective energy $E_{\text{intr}} = M_Q \dfrac{2\Pi^2}{\beta}$, 3) potential energy $E_q = M_Q \dfrac{\alpha Q^2}{2}$, 4) kinetic energy of collective flow $E_{\text{kin}} = M_Q \dfrac{\dot{Q}^2}{2}$

time when the regime of the motion has an elastic character. Only after the installation of the plastic (viscose) regime at larger time these two components of the energy disappear, and the temperature is well defined by the potential energy of the system.

5. Fluctuations in Classical Elastoplastic Systems

Equations (1), (2) chosen here to describe an elastoplastic system give a very limited information of such a complex many-body physical object: the time evolution of some collective degrees of freedom of it. The dissipation of the collective energy signifies an interaction of these degrees of freedom with the numerous others. The description given by these equations may be called «macroscopic» as, for example, one calls the description in terms of the friction force of a massive (Brownian) particle propagation through the liquid [7]. When the size of the Brownian particle decreases, the quality of such a description diminishes. To learn more about the motion of a «small» system, one must pass from the «macroscopic» to «microscopic» description. In this section we make a step towards it for our «elastoplastic system» following the standard techniques established for the study of the Brownian motion.

Let us introduce a «microscopic» Hamiltonian

$$H_{\text{micr}} = H_{\text{coll}} - ZF_\xi + H_\xi, \tag{15}$$

where H_{coll} is the collective Hamiltonian of Eq.(11) and H_ξ is an «intrinsic» Hamiltonian operating in the space of intrinsic co-ordinates and momenta (ξ). With the chosen form of the coupling term ($-ZF_\xi$), the Hamiltonian H_{micr} leads to the same equations of motion as in Eqs.(12) and in the first of Eqs.(13). The second of the latter equations takes the form:

$$\dot{P}_Z = F_\xi. \tag{16}$$

where the quantity $\sigma^2 = \sigma_0^2 T$ measures the amplitude of perturbations introduced by the random force $\delta F(t)$. Then

$$\overline{(\delta Q(t))^2} = \left(\frac{\sigma_0\beta}{2M_Q}\right)^2 \int_0^t dt' G(t-t')^2 T(t'). \tag{23}$$

The value of parameter σ in Eq.(22) may be determined from the thermal equilibrium conditions between the collective and intrinsic motions, just as in the case of the Brownian motion. We remind that the last term in the expression (14) for the collective energy represents intrinsic degrees of freedom. The fluctuating force $\delta F(t)$ contributes, in particular a term

$$\overline{\delta E^{Q}_{\text{coll}}(t)} = \frac{M_Q}{2}\left(\overline{\delta\dot{Q}^2} + \alpha\overline{\delta Q^2}\right) \tag{24}$$

to the part of the collective energy which represents the «Q» degree of freedom in the collective Hamiltonian in Eq.(11).

In the case of the thermal equilibrium with a medium (i.e., with the ensemble of intrinsic degrees of freedom) at the temperature T (which is here supposed to be sufficiently high) the system must accumulate on the average an energy $\overline{\delta E^{Q}_{\text{coll}}(T)} = kT$, where k is the Boltzmann constant. Equating $\delta E^{Q}_{\text{coll}}(t \to \infty) \equiv (\sigma)^2\Delta$ with the latter expression one finds:

$$\sigma^2 = \frac{kT}{\Delta}. \tag{25}$$

Suggesting a Gauss distribution of the fluctuating quantities one may find the probability of deviations of these observables from their positions on the macroscopic trajectory [7]. In systems with strongly accentuated elastoplasticity, where $\beta >> \alpha$ and $\sqrt{\beta} >> 1/2\tau$, the quantity $\overline{\delta Q^2}$ regarded as a function of time lags behind $\overline{\delta\dot{Q}^2}$ in its saturation properties during the time $0 < t \le \tau\beta/\alpha$ (Fig.4). As a result during this time interval the averaged fluctuation energy in Eq.(24) remains lower than kT. One may say that the temperature determining the statistical properties of quantities depending on Q and $\dot{Q}$ and defined as

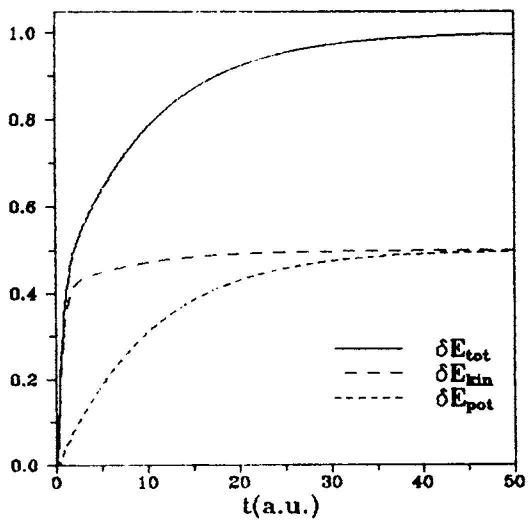

Fig.4. Time dependence of averaged fluctuating collective energy in «Q» channel $\overline{\delta E_{\text{coll}}(t)}$ devided by its asymptotic value $\overline{\delta E_{\text{coll}}(t \to \infty)}$ and of the functions $\delta E_{\text{pot}} = \sqrt{\overline{\delta Q^2}}$ and $\delta E_{\text{kin}} = \sqrt{\overline{\delta \dot{Q}^2}}$